CHAPTER IV.

44. Antiquity of Masonry Aqueducts. — Masonry aqueducts, either solid or with open arches, were not first constructed by the city of Rome; their origin was much farther back in antiquity than that. The Greeks at least used them before the Roman engineers, and it is not unlikely that the latter drew their original ideas from the former, if indeed they were not instructed by them. Nor during the times of the Romans was the construction of aqueducts confined to Rome. Wherever Roman colonies were created it would appear that vast sums were expended in the construction of aqueducts for the purpose of suitably supplying cities with water. Such constructions are found at many points in Spain, France, and other countries which were in ancient times Roman colonies. It is probable that there are not less than one hundred, and perhaps many more, of such structures in existence at the present time.

45. Pont du Gard.—Among the more prominent aqueducts constructed during the old Roman period and outside of Italy were the Pont du Gard at Nismes in the south of France, and those at Segovia and Tarragona in Spain. The Pont du Gard has three tiers of arches with a single channel at the top. The greatest height above the river Gardon is about 180 feet, and the length of the structure along the second tier of arches is 885 feet. The arches in the lowest tier are 51 feet, 63 feet, and 80.5 feet in span, while the arches in the highest tier are uniformly 15 feet 9 inches in span. The thickness of the masonry at the top of the structure from face to face is 11 feet 9 inches, and 20 feet 9 inches at the lower tier of arches, the thickness at the intermediate tier being 15 feet.

The largest arch has a depth of keystone of 5 feet 3 inches, while the other arches of the lower tier have a depth of keystone of 5 feet. The depth of the ring-stones of the small upper arches is 2 feet 7 inches. This structure forms a sort of composite construction, the lower arches constituting four separate arch-rings placed side by side, making a total thickness of 20 feet 9 inches. The intermediate arches consist of three similar series of narrow arches placed side by side, but the masonry of the upper tier is continuous throughout from face to face. The three and four parallel series of arches of the middle and lowest tiers are in no way bonded or connected with each other. There is no cementing material in any of the arch-rings, but cement mortar was used in rubble masonry or concrete around the channel through which the water flowed above the upper tier of small arches. This structure is supposed to have been built between the years 31 B.C. and 14 A.D.

46. Aqueducts at Segovia, Metz, and Other Places. — The Segovia aqueduct was built by the emperor Trajan about A.D. 100–115. It is built without mortar, and has 109 arches, but 30 are modern, being reproductions of the old. It has a length of over 2400 feet, and in places its height is about 100 feet. The old Tarragona aqueduct is built with two series of arches, 25 being in the upper series and 11 in the lower. It is 876 feet long and has a maximum height of over 80 feet. At Mayence there are ruins of an aqueduct about 16,000 feet long. In Dacia, Africa, and Greece there are other similar ruins. Near Metz are the remains of a large old Roman aqueduct. It consisted of a single row of arches, and had no features of particular prominence. This latter observation, however, could not be made of one of the bridges in the aqueduct at Antioch. Although the masonry and design of this latter structure were crude, its greatest height is 200 feet, and its length 700 feet. The lower portion of this structure was a solid wall with the exception of two openings, the arches extending in a single row along its upper portion. On the island of Mytilene are the ruins of another old aqueduct about 500 feet long, with a maximum height of about 80 feet.

The building of these remarkable aqueducts was practised at least down to the later periods of the Roman empire, that of Pyrgos, near Constantinople,—built not earlier than the tenth century,—being an excellent example. It consists of two branches at right angles to each other. The greater branch is 670 feet long, and its greatest height 106 feet. There are three tiers of arches, the two upper being of semicircular and the lower of Gothic outline. The number in each tier for a given height is the same, but with an increasing length of span in rising from the lowest to the highest tier. Thus the highest tier of piers is the lightest, relieving the top of the structure of weight. The lowest row of piers is reinforced by counterforts or buttresses. At the top of the structure the width or thickness is 11 feet, but the thickness increases uniformly to 21 feet at the bottom. The smaller branch of the aqueduct is 300 feet long, and was built with twelve semicircular arches.

47. **Tunnels.**—The construction of tunnels, especially in connection with the building of aqueducts, constituting a branch of engineering procedure, was frequently practised by the ancient nations. Large tunnel-works were executed many times by the ancient Greeks and Romans. It would seem that the Greeks were the instructors of the Romans in this line of engineering operations. As early as B.C. 625 we are told that the Greek engineer Eupalinus constructed a tunnel 8 feet broad, 8 feet high, and 4200 feet long, through which was built a channel for carrying water to the city of Athens.

Sixty-five years later a similar work was constructed for the same Grecian city. Indeed it appears that tunnels were constructed in the time of the earliest history of aqueducts built to supply ancient Greek and Roman cities with water.

It is certain that at the beginning of the Christian era tunnelling processes were well known among the Romans. Vitruvius writes, in speaking of the construction of aqueducts, in Chapter VII of the Eighth Book: "If hills intervene between the city wall and spring head, tunnels underground must be made, preserving the fall above assigned; if the ground cut through be sandstone or stone, the channel may be cut therein; but if the soil be earth or gravel, side walls must be built, and an arch turned over, and through this the water may be conducted. The distance between the shafts over the tunnelled part is to be 120 feet."

The Romans pierced rock in their tunnel-work, not only by chiselling, but sometimes by building fire against the rock so as to heat it as hot as possible. The heated rock was then drenched with cold water, so that it might be cracked and disintegrated to as great an extent as practicable. According to Pliny vinegar was used instead of water in some cases, under the impression that it was more efficacious.

One of the methods mentioned by Vitruvius is plainly "the cut and cover" procedure of the present day. In Duruy's history of Rome a tunnel over three miles long is mentioned on a line of an aqueduct at Antibes in France, as well as another constructed to drain Lake Fucinus in Italy, about A.D. 50. It is there stated that the latter required eleven years' labor of 30,000 men to build a rock tunnel with a section of 86 to 96 square feet 18,000 feet long.

Roman water-pipe made of bored-out blocks of stone.

Lanciani, in his "Ancient Rome," states that about A.D. 152 a Roman engineer (Nonius Datus) began the construction of a tunnel in Algeria, and after having carefully laid out the axis of the tunnel across the ridge "by surveying, and taking the levels of the mountains," left the progress of the work in the hands of the contractor and his workmen. After the rather long absence from such a work of four years he was called back by the Roman governor to ascertain why the two opposite sections of the tunnel, as constructed, would not meet, and to take the requisite measures for the completion of the work through which water was to be conducted to Saldæ in a suitable channel. He explains that there should have been no difficulty, and that the failure of the two headings to meet was due to the negligence of the contractor and his assistant, whom he states " had committed blunder upon blunder," although he writes, "As always happens in these cases, the fault was attributed to the engineer." He solved the problem by connecting the two approximately parallel tunnels by a transverse tunnel, so that water was finally brought to the city of Saldæ.

The art of tunnel construction has been one of the most widely practised branches of Civil Engineering from the times of the ancient Assyrians, Egyptians, Greeks, Romans, and other ancient nations down to the present.

48. Ostia, the Harbor of Rome.—The capacity of the ancient Romans to build harbor-works is shown by what they did at Ostia, which was then at the mouth of the Tiber, but is now not less than four miles inland from the present shore-line. At the Ostia mouth of the river the present annual average advance seaward is not less than 30 feet, and at the Fiumicino mouth about one third of that amount.

The ancient port of Ostia is supposed to have been founded during the reign of the fourth king Ancus Marcius, but it attained its period of greatest importance during the reign of Claudius and Trajanus. At that time the fertile portions of the Campania had been so largely taken up by the country-places of the wealthy Romans that it was no longer possible for the peasantry to cultivate sufficient ground to yield the grain required by the home market of the Romans. Large fleets were consequently engaged in the foreign grain-trade of Rome. The wheat and other grain required in great quantities was grown mostly in Egypt, although Carthage and other countries supplied large amounts. The great fleets occupied in this trade made ancient Ostia their

Roman port. At the present time it has no inhabitants, but is a group of complete ruins, with its streets of tombs, baths, palaces, and temples, deeply covered with the accumulations of many centuries. Enough excavations have been made along the shores of the Tiber at this point to show that the river was bordered with continuous and substantial masonry quays, flanked

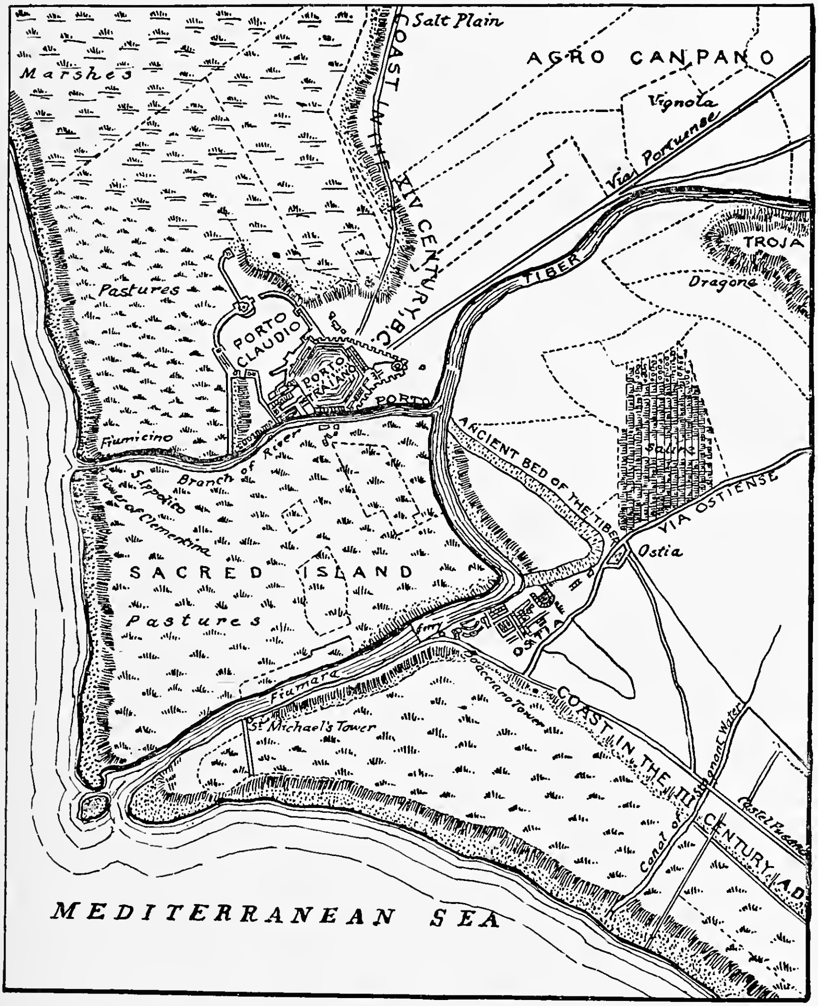


FIG. 13.—Plan of Ostia and Porto.

on the land side by successions of great warehouses, obviously designed to receive grain, wine, oil, and other products of the time. The entrance to this harbor was difficult, as the mouth of the river was shallow, with bars apparently obstructing its approach. There were no jetties, or other seaward works for the protection of vessels desiring to make the harbor. It is stated that during one storm nearly or quite two hundred vessels were destroyed while they were actually in the harbor.

49. **Harbors of Claudius and Trajan.**—The difficulty in entering the mouth of the Tiber prompted the emperor Claudius to construct another harbor to accommodate the vast commerce then centring at the port of Rome. Instead of increasing the capacity of Ostia and opening the mouth of the river by deepening it, he constructed a new harbor on what was then the seashore, a short distance from Ostia, and connected it with the Tiber by a canal, the extension of which by the natural forces of the river has become the Fiumicino, the only present navigable entrance to the river. This harbor was enclosed by two walls stretching out from the shore, and converging on the sea side to a suitable opening left for the entrance of ships. The superficial area of this harbor was about 175 acres, but it became insufficient during the time of Trajan. He then proceeded to excavate inland a hexagonal harbor with a superficial area of about 100 acres, which was connected both with the harbor of Claudius and the canal connecting the latter with the Tiber. These harbor-works were elaborate in their fittings for the accommodation of ships, and were built most substantially of masonry. They showed that at least in some branches of harbor-work the old Romans were as good engineers as in the construction of aqueducts, bridges, and other internal public works. The harbors at ancient Ostia, including those of Claudius and Trajan, were not the only works of their class constructed by the Romans, but they are sufficient to show as great advancement in harbor and dock work as in other lines of engineering.

These harbors were practically defenceless and exposed to the incursions of pirates, which came to be frequently and successfully made in the days of the declining power of Rome. It was therefore rather early in the Christian era that these attacks discouraged, and ultimately drove away, first, the maritime business of the Romans and, subsequently, all the inhabitants of these ports, leaving the pillaged remnants of the vast harbor-works, warehouses, palaces, temples, and other buildings in the ruined condition in which they are now found.

CHAPTER V.

50. Ancient Engineering Science. — The state of what may be called the philosophy or science of engineering construction in ancient Rome is admirably illustrated by the work on Architecture by Marcus Vitruvius Pollio, who is ordinarily known as Vitruvius, and who wrote probably a little more than two thousand years ago. He calls himself an architect, and his work is a classic in that profession of which he claims to be a member. Although much of his work was purely architectural, a great portion of it, on the other hand, was not architecture as we now know it, but civil engineering in the best sense of the term. It must be remembered, therefore, that what is here written applies to that large portion of his work which is purely civil engineering.

It will be seen that although he understood really little or nothing about the science of civil engineering as we now comprehend it, he perceived many of the general and fundamental principles of the best practice of that profession and frequently applied them in a manner which would do credit to a modern civil engineer. He not only laid down axioms to govern the design of civil-engineering structures and machinery for the transmission of power, but he also set forth many considerations bearing upon public and private health and the practice of sanitary engineering in a way that was highly creditable to the state of scientific knowledge in his day. Speaking of the general qualifications of an architect, remembering that that word as he understood it includes the civil engineer, he states: "An architect should be ingenious, and apt in the acquisition of knowledge; . . . he should be a good writer, a skilful draughtsman, versed in geometry and optics, expert at figures, acquainted with history, informed on the principles of natural and moral philosophy, somewhat of a musician, not ignorant of the sciences both of law and physics, nor of the motions, laws, and relations to each other of the heavenly bodies." Again he adds: "Moral philosophy will teach the architect to be above meanness in his dealings and to avoid arrogance; it will make him just, compliant, and faithful to his employer; and, what is of the highest importance, it will prevent avarice gaining an ascendency over him; for he should not be occupied with the thoughts of filling his coffers, nor with the desire of grasping everything in the shape of gain, but by the gravity of his manners and a good character should be careful to preserve his dignity."

These quaint statements of the desirable qualities of a professional man are worthy to be considered rules of good professional living at this time fully as much as they were in the days of old Rome. His esteem for his profession was evidently high, but not higher than the value which every civil engineer should put upon his professional-life. The need of a general education for a civil engineer is greater now even than in his day, although musical accomplishments need not be considered as essential in modern engineering practice. That qualification, it is interesting to observe in passing, was inserted by Vitruvius in order to illustrate the wide range of engineering practice in those days when the architect-engineer was called upon, among other things, to construct catapults and other engines of war, in which a nice adjustment of gut ropes was determined by the musical tones emitted under the desired tension.

51. Ancient Views of the Physical Properties of Materials.—When it is remembered that the chemical constitution of materials used in engineering was absolutely unknown, that no quantitative determination of physical qualities had been made, and that the first correct conception of engineering science had yet to be acquired, it is a matter of wonder that there had been attained the engineering development evidenced both by ancient writings like those of Vitruvius and great engineering works like those of Rome, in the Babylonian Plain and in Egypt. In discussing the problem of water-supply, he mentions that certain learned ancients, "physiologists and philosophers, maintained that there are four elements—air, fire, water, and earth—and that their mixture, according to the difference of the species, formed a natural mode of different qualities. We must recollect that not only from these elements are all things generated, but that they can neither be nourished nor grow without their assistance.'' This view of the construction of material things was not conducive to a clear comprehension of those physical laws which lie at the foundation of engineering science, and it is absolutely essential that these elementary considerations be kept constantly in view in considering the engineering attainments of the Romans and other ancient peoples.

52. Roman Civil Engineers Searching for Water.—In ancient times, as at present, it was very important in many cases to know where to look for water, and how to make what might promise to be a successful search for it. Vitruvius states that the sources of water for a supply may easily be found ''if the springs are open and flowing above ground." If the sources are not so evident, but are more obscure, he recommends that ''before sunrise one must lie down prostrate in the spot where he seeks to find it, and, with his chin placed on the ground and fixed, look around the place; for, the chin being fixed, the eye cannot range upwards further than it ought and is confined to the level of the place. Then where the vapors are seen curling together and rising into the air, there dig, because those appearances are not discovered in dry places." This method of discovering water-supply would be considered by modern engineers at least somewhat awkward as well as damp and disagreeable in the early morning hours. It is not more fantastic, however, or less philosophical than the use of the divining-rod, which has been practised in modern times as well as ancient, and is used even in some country districts at the present time.

Vitruvius does not forget that the local features, including both those of soil and of an artificial character, may affect the quality of the water and possibly make it dangerous. He, therefore, sets forth general directions by which good potable water may be found and that of a dangerous nature avoided. The necessity of distinguishing between good and bad water was as present to his mind and to the minds of the old Roman engineers as to civil engineers of the present day, but the means for making a successful discrimination were crude and obviously faulty, and very often unsuccessful. He set forth, what is well known, that rain-water when collected from an uncontaminated atmosphere is most wholesome, but proceeds to give reasons which would not now be considered in the highest degree scientific.

In Chapter V of his Eighth Book there are described some "means of judging water" so quaint and amusing that they may now well be quoted even though no civil engineer would be bold enough to cite them in modern hydraulic practice. He says: "If it be of an open and running stream, before we lay it on, the shape of the limbs of the inhabitants of the neighborhood should be looked to and considered. If they are strongly formed, of fresh color, with sound legs and without blear eyes, the supply is of good quality." At another point he comes rather closely to our modern requirements which look to the exclusion of minute and elementary vegetable growths, when he says: "Moreover, if the water itself, when in the spring, is limpid and transparent, and the places over which it runs do not generate moss, nor reeds, nor other filth be near it, everything about it having a clean appearance, it will be manifest by these signs that such water is light and exceedingly wholesome."

53. Locating and Designing Conduits.— In treating of the manner of conducting water in pipes or other conduits, he adverts to the necessity of accurate levelling and the instruments that were used for that purpose. The three instruments which he mentions as being used are called the dioptra, the level (*libra aquaria*), and the chorobates, the latter consisting of a rod about 20 feet in length, having two legs at its extremities of equal length and at right angles to it. Cross-pieces were fastened between the rod and the legs with vertical lines accurately marked on them. These vertical lines were placed in a truly vertical position by means of plumb-lines so that the top of the rod was perfectly level, and the work could thus be made level in reference to it.

In Rome the water was generally conducted either by means of open channels, usually built in masonry for the purpose, or in lead pipes, or in "earthen tubes." Vitruvius states that the open channels should be as solid as possible, and have a fall of not less than one half a foot in 100 feet. The open channels were covered with an arch top, so that the sun might be kept from striking the water. After bringing the water to the city it was divided into three parts. One was for the supply of pools and fountains, another for the supply of baths, and a third for the supply of private houses. A charge was made for the use of water for the pools, fountains, and baths, and in this way a yearly revenue was obtained. A further charge was also made for the water used in private houses, the revenue from which was applied for the maintenance of the aqueduct which supplied the water. The treatment to be given to the different soils, rocks, and other materials through which the conduit was built which brought the supply to Rome is duly set forth by Vitruvius, and he describes the conditions under which tunnels were constructed. He also described the methods of classifying the lead pipes through which water was conducted from the reservoirs to the various points in the city after stating that they must be made in lengths of not less than 10 feet. The sheets of lead employed in the manufacture of the pipes he describes as ranging in width from 5 inches to 100 inches. The diameter of the pipe would obviously equal very closely the width of the sheet divided by the ratio between the circumference and the diameter of the corresponding circle.

54. **Siphons.**—He speaks of passing valleys in the construction of the conduits by means of what we now call siphons, and prescribes a method for relieving it of the accumulated air. In speaking of earthen tubes or pipes he says that they are to be provided not less than 2 inches thick and "tongued at one end so that they may fit into one another," the joints being coated with quicklime and oil. He further observes that water conducted through earthen pipes is more wholesome than that through lead, and that water conveyed in lead must be injurious because from it white lead is obtained, which is said to be injurious to the human system. Indeed the effects of lead-poisoning were recognized in those early days, and its avoidance was attempted. In the digging of wells he wisely states that "the utmost ingenuity and discrimination" must be used in the examination of the conditions under which wells were to be dug. He also appreciated the advantage of sedimentation, for he advises

that reservoirs be made in compartments so that, as the water flows from one to another, sedimentation may take place and the water be made more wholesome.

55. Healthful Sites for Cities.—In the location of cities, as well as of private residences, Vitruvius lays down the general principle that the greatest care should be taken to select sites which are healthy and subject only to clean and sanitary surroundings. Marshy places and those subject to fogs, especially those "charged with the exhalations of the fenny animals," are to be avoided. Apparently this reference to "fenny animals" may have beneath it the fundamental idea of bacteria, but that is not certain. The main point of all these directions for the securing of sanitary conditions of living is that, so far as his technical knowledge permitted him to go, he insists on the same class of wholesome conditions that would be prescribed by a modern sanitary engineer.

56. Foundations of Structures.—Similarly in Chapter V of his First Book, on "Foundations of Walls and Towers," Vitruvius shows a realization of the principal conditions needful and requisite for the suitable founding of heavy buildings. After a sanitary site for a city is determined and one that can be put in communication with other people "by good roads, and river or sea navigation for the transportation of merchandise," he proceeds to state that "foundations should be carried down to solid bottom, if such can be found, and that they should be built thereon of such thickness as may be necessary for the proper support of that part of the wall standing above the natural level of the ground. They should be of the soundest workmanship, and materials of greater thickness than the walls above." Again, in speaking of the foundations supporting columns, he states: "The intervals between the foundations brought up under the columns should be either rammed down hard, or arched, so as to prevent the foundation-piers from swerving. If solid ground cannot be come to, and the ground be loose or marshy, the place must be excavated, cleared, and either alder, olive, or oak piles, previously charred, must be driven with a machine as close to each other as possible and the intervals between the piles filled with ashes. The heaviest foundations may be laid on such a base." It is thus seen that pile foundations were used by the Romans, and that the piles were driven with a machine. It would be difficult to give sounder general rules of practice even after more than two thousand years' additional experience.

57. Pozzuolana and Sand.—Of all the materials which were useful to the Romans in their various classes of construction, including the foundations of roads, "pozzuolana" must have been the most useful, and that which contributed more to the development of successful construction in Rome than any other single agent. Vitruvius speaks of it frequently and gives rules not only for the use of it in the production of mortar and concrete, but also lays down at considerable length the treatment which should be given to lime in order to produce the best results. It was common, according to his statements, to use two measures of "pozzuolana" with one of lime in order to obtain a suitable cementing material. This mixture was used in varying proportions with sand and gravel or broken stone to produce concrete. He describes the various grades of sands to be found about Rome and the manner of using them. The statement is made that sand should be free of earth and that the best of it was such as to yield a "grating sound" when "rubbed between the fingers." This is certainly a good engineering test of sand. He prefers pit-sand to either river- or sea-sand; indeed throughout all his directions regarding this particular class of construction his rules might be used at the present time with perfect propriety.

58. Lime Mortar.—The old Romans had also discovered the advisability of allowing lime to stand for a considerable period of time after slaking. This insured the slaking of all those small portions which were possibly a little hydraulic and therefore slaked very slowly. He prescribes as a good proportion two parts of sand to one of lime, and also mentions the proportion of three to one. He attempts to explain the setting, as we term it, of lime, but his explanation in obscure terms, involving qualities of the elements of fire and air, is not very satisfactory.

59. Roman Bricks according to Vitruvius.—As is well known, the Romans were good brick-makers, and they were well aware that bricks made from "ductile and cohesive" "red or white chalky" earth were far preferable to those made of more gravelly or sandy clay. The Roman bricks were both sun-dried and kiln-burned.

60. Roman Timber.—Timber was a material much used by the Romans, and the greater part of that which they used probably was grown in Italy, although considerable quantities were imported from other localities. Vitruvius writes in considerable detail concerning the selection of timber while standing, as well as in reference to its treatment before being used in structures. Like every material used by the old Romans in construction, the various kinds and qualities of timber received careful study from them, and they were by no means novices in the art of producing the best results from those kinds of timber with which they were familiar.

61. The Rules of Vitruvius for Harbors.—In Chapter XII of his Fifth Book Vitruvius lays down certain general rules for the selection and formation of harbors, and it is known that the Romans were familiar with elaborate and effective harbor construction, as is shown by that at Ostia. He appreciates that a natural harbor is one which has "rocks or long promontories jutting out, which from the shape of the place form curves or angles," and that in such places "nothing more is necessary than to construct portices and arsenals around them, or passages to the markets." He then proceeds to state that if such a natural formation is not to be found, and that if "on one side there is a more proper shore than on the other, by means of building or of heaps of stones, a projection is run out, and in this the enclosures of harbors are formed." He then proceeds to explain how "pozzuolana" and lime, in the proportion of two of the former to one of the latter, are used in subaqueous construction. He also prescribed a mode of building a masonry wall up from the bottom of an excavation made within what we should call a coffer-dam, formed, among other things, "of oaken piles tied together with chain pieces." The Romans knew well how to select harbors and how to construct in an effective manner the artificial works connected with them, although it appears that the effects of tidal and river currents in estuaries were neither well understood in themselves nor in their transporting power of the solid material which those currents eroded.

62. The Thrusts of Arches and Earth; Retaining-walls and Pavements.—Although the Romans possessed little or no knowledge of analytical mechanics they attained to some good qualitative mechanical conceptions. Among other things they understood fairly well the general character of the thrust of an arch and the tendency of the earth to overthrow a retaining-wall. They knew that a massive abutment was needed to receive safely the thrust of an arch, and they counterforted or buttressed retaining-walls in order to hold them firmly in place. They also realized the danger of wet earth pressing against a retaining-wall, and even made a series of offsets or teeth on the inside of the wall on which the earth rested in order to aid in holding the wall in place. Vitruvius recommends as a safeguard against the pressure of earth wet by winter rains that "the thickness of the wall must be proportioned to the weight of earth against it," and that counterforts or buttresses be employed "at a distance from each other equal to the height of the foundations, and of the same width as the foundations," the projections at the bottom being equal in thickness to that of the wall, and diminishing toward the top.

He gives in considerable detail instructions for the forming of pavements and stucco work, so many examples of which are still existing in Rome. These rules are in many respects precisely the same as would govern the construction of similar work at the present time. There are also described in a general way the methods of producing white and red lead, as pigments of paints, and a considerable number of other pigments of different colors.

63. The Professional Spirit of Vitruvius.—It is evident, from many passages in the writings of this Roman architect-engineer, that the ways of the professional men in old Rome were not always such as led to his peace of mind. Vitruvius utters bitter complaints which show that he did not consider purely professional knowledge and service to be adequately recognized or appreciated by his countrymen. He writes that in the city of Ephesus an ancient law provided that if the cost of a given work completed under the plans and specifications of an architect did not exceed the estimate, he was commended "with decrees and honors," but if the cost exceeded the estimate with 25 per cent added thereto, he "was required to pay that excess out of his own pocket." Then he exclaims, "Would to God that such a law existed among the Roman people, not only in respect to their public but also to their private buildings, for then the unskilful could not commit their depredations with impunity, and those who were the most skilful in the intricacies of the art would follow the profession!"

64. Mechanical Appliances of the Ancients.—It is well known that the ancients possessed at least some simple types of machines, for the reason that they raised many great stones to a considerable height in completed works after having transported them great distances from the quarries whence they were taken. Undoubtedly these machines were of a simple and crude character and were made effective largely by the power of great numbers of men. We are not acquainted with all the details of these machines, although the general types are fairly well known. The elementary machines, including the lever, the inclined plane, the pulley, and the screw, which is only an application of the inclined plane, were all used not only by the Romans, but probably by every civilized ancient nation. Vitruvius describes a considerable number of these machines, and from his descriptions it is clear that they had wide application in the structural works of the Romans. The block and fall, as we term the pulley at the present time, was a common machine in the plant of a Roman constructor, as were also various modifications and applications of the lever, the roller, and the inclined plane.

65. Unlimited Forces and Time.—It is neither surprising nor very remarkable that with the use of these simple machines, aided by a practically unlimited number of men, the necessary raising or other movement of heavy weights was accomplished by the Romans and other ancient peoples. It is to be borne in mind that the element of time was of far less consequence in those days than at present, and that the rate of progress made in the construction of most if not all ancient engineering works was what we should consider intolerably slow.

PART II.

BRIDGES.

CHAPTER VI.

66. Introductory.—Although the bridge structures of to-day serve the same general purposes as those served by the most ancient structures, they are very different engineering products. It is not long, in comparison with the historic and prehistoric periods during which bridges have been built, since the science of mechanics has been sufficiently developed to make bridge design a rational procedure; and it is scarcely more than a century since the principles of mechanics were first applied to the design of bridge structures in such a way as to determine even approximately the amount of stress produced in any member by the imposed load. Naturally the first efforts made toward a truly rational bridge design were in fact simple and crude and only loosely approximate in their results. Probably the first analytic treatment of bridges was given to the design of arches in masonry and then in cast iron. As the action of forces in structures became better known through the development of mechanical science, the applications of the latter became less crude and approximate and the approach to the refined accuracy of the present day was begun.

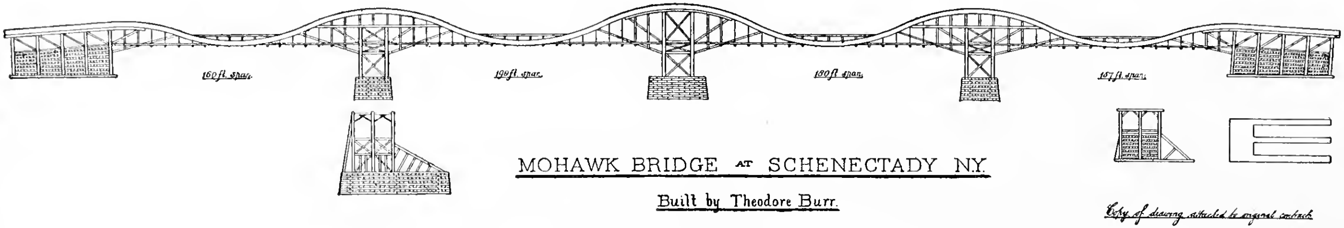
67. First Cast-iron Arch.—These older structures, nearly all of them arches or more or less related to the arch, first appeared in cast iron in the latter part of the eighteenth century, when nothing like an accurate analysis of forces developed by the application of a given load was known. The first cast-iron arch was erected over the Severn in England near Coalbrookdale in the year 1779. This bridge had a span of 100 feet, and the under surface of the arch or soffit at the crown was 45 feet above the points at the abutment from which the arch sprang, or, as civil engineers put it, the arch had a span of 100 feet and a rise or versine of 45 feet. Other cast-iron arches were built in England soon after.

68. Early Timber Bridges in America.—Timber bridges have been built since the earliest historic periods and even earlier, but the widest and boldest applications of timber to bridge structures have been made in this country, beginning near the end of the eighteenth century and running to the middle of the nineteenth century, when timber began to be displaced by iron. Timber bridges and those of combined iron and timber are built to some extent even at the present day, but the most extended work of this class is to be found in the period just named.

In 1660 what was called the "Great Bridge" was built across the Charles River near Boston, and was a structure on piles. Other similar structures followed, but the first long-span timber bridge, where genuine bridge trussing or framing was used, appears to have been completed in 1792, when Colonel William P. Riddle constructed the Amoskeag Bridge across the Merrimac River at Manchester, N. H., in six spans of a little over 92 feet from centre to centre of piers. From that time timber bridges, mostly on the combined arch and truss principle, were built, many of them examples of remarkably excellent engineering structures for their day. Among these the most prominent were the Bellows Falls Bridge, in two spans of 184 feet each from centre to centre of piers, over the Connecticut River, built in 1785–92 by Colonel Enoch Hale; the Essex-Merrimac Bridge over the Merrimac River, three miles above Newburyport, Mass., built by Timothy Palmer in 1792, consisting actually of two bridges with Deer Island between them, the principal feature of each being a kind of arched truss of 160 feet span on one side of the island and 113 feet span on the other; the Piscataqua Bridge, seven miles above Portsmouth, N. H., in which a "stupendous arch of 244 feet cord is allowed to be a masterly piece of architecture, planned and built by the ingenious Timothy Palmer of Newburyport, Mass.," in 1794; the so-called "Permanent Bridge" over the Schuylkill River at Philadelphia, built in 1804–06 in two arches of 150 feet and one of 195 feet, all in the clear, after the design of Timothy Palmer; the Waterford Bridge over the Hudson River, built in 1804 by Theodore Burr, in four combined arch and truss spans, one of 154 feet, one of 161 feet, one of 176 feet, and the fourth of 180 feet, all in the clear; the Trenton Bridge, built in 1804–06 over the Delaware River at Trenton, N. J., by Theodore Burr, in five arch spans of the bowstring type, ranging from 161 feet to 203 feet in the clear; a remarkable kind of wooden suspension bridge built by Theodore Burr in 1808 across the Mohawk River at Schnectady, N. Y., in spans ranging in length from 157 feet to 190 feet; the Susquehanna Bridge at Harrisburg, Pa., built by Theodore Burr in 1812–16 in twelve spans of about 210 feet each; the so-called Colossus Bridge, built in 1812 by Lewis Wernwag over the Schuylkill River at Fairmount, Pa., with a clear span of 340 feet 3¾ inches; the New Hope Bridge, built in 1814 over the Delaware River, in six 175 feet combined arch and truss spans, and a considerable number of others built by the same engineer.

Some of these wooden bridges, like those at Easton, Pa., and at Waterford, N. Y., remained in use for over ninety years with only ordinary repairs and with nearly all of the timber in good condition. In such cases the arches and trusses have been housed and covered with boards, so as to make what has been commonly called a covered bridge. The curious timber suspension bridge built by Theodore Burr at Schenectady was used twenty years as originally built, but its excessive deflection under loads made it necessary to build up a pier under the middle of each span so as to support the bridge structure at those points. These bridges were all constructed to carry highway traffic, but timber bridges to carry railroad traffic were subsequently built on similar plans, except that Burr's plan of wooden suspension bridge at Schenectady was never repeated.

69. Town Lattice Bridge.—A later type of timber bridge which was most extensively used in this country was invented by Ithiel Town in January, 1820, which was known as the Town



MOHAWK BRIDGE AT SCHENECTADY N.Y.

Built by Theodore Burr.

Copy of drawing attached to original contract.

FIG. 1.

PATENT BRIDGE "COLOSSUS"

Across the River Schuylkill at Philadelphia.

Single Arch 340 feet 3 & 3/4 inches.

Built by Lewis Wernwag.

FIG. 2.

lattice bridge. This timber bridge was among those used for railroad structures. As shown by the plan it was composed of a close timber lattice, heavy plank being used as the lattice members, and they were all joined by wooden pins at their intersections. This type of timber structure was comparatively common not longer ago than twenty-five years, and probably some structures of its kind are still in use. The close latticework with its many pinned intersections made a very safe and strong framework, and it enjoyed deserved popularity. It was the forerunner in timber of the modern all-riveted iron and steel lattice truss. It is of sufficient significance to state, in connection with the Town lattice, that its inventor claimed that his trusses could be made of wrought or cast iron as well as timber. In many cases timber arches were combined with them.

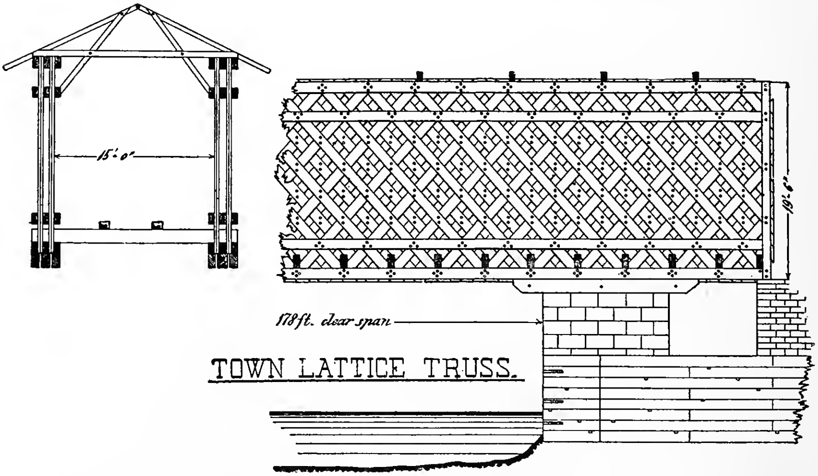


FIG. 3.

70. Howe Truss.—The next distinct advance made in the development of bridge construction in the United States was made by brevet Lieutenant-Colonel Long of the Corps of Engineers, U.S.A., in 1830–39, and by William Howe, who patented the bridge known as the Howe truss, although the structure more lately known under that name is a modification of Howe's original truss. Long's truss was entirely of timber, including the keys, pins, or treenails required, and it was frequently built

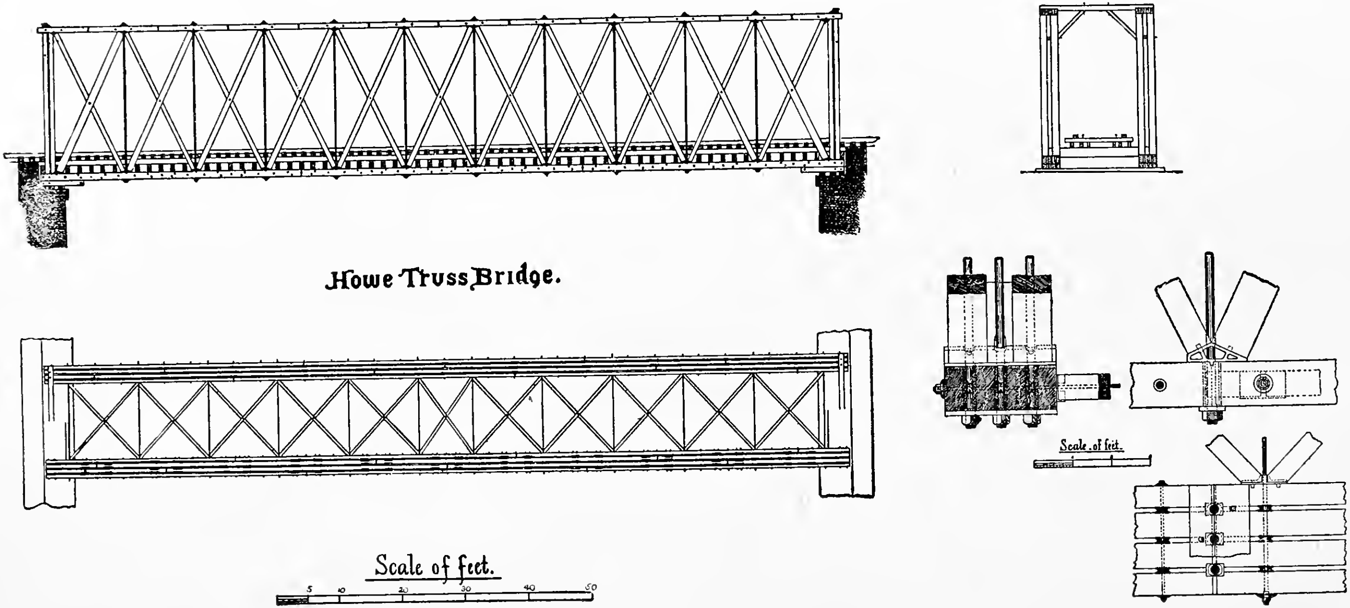


FIG. 4.

in combination with the wooden arch. The truss was considerably used, but it was not sufficiently popular to remain in use.

The Howe truss was not an all-wooden bridge. The top and bottom horizontal members, known as "chords," the inclined braces between them and the vertical end braces, all connecting the two chords, were of timber, and they were bolted at all intersections; but the vertical braces were of round iron with screw ends. These rods extended through both chords and received nuts at both ends pressing on cast-iron washers through which the rods extended. These wrought-iron round rods were in groups at each panel-point, numbering as many as existing stresses required. The ends of the timber braces abutted against cast-iron joint-boxes. The railroad floor was carried on heavy timber ties running entirely across the bridge and resting upon the lower chord members. It was a structure simple in character, easily framed, and of materials readily secured. It was also easily erected and could quickly be constructed for any reasonable length of span. It possessed so many merits that it became widely adopted and is used in modified form at the present day, particularly on lines where the first cost of construction must be kept as low as possible. The large amount of timber in it and the simple character of its wrought-iron or steel members greatly reduces its first cost.

71. Pratt Truss.—In 1844 the two Pratts, Thomas W. and Caleb, patented the truss, largely of timber, which has since been perpetuated in form by probably the largest number of iron and steel spans ever constructed on a single type. The original Pratt trusses had timber upper and lower chords, but the vertical braces were also made of timber instead of iron, while the inclined braces were of round wrought iron with screw ends, the reverse of the web arrangement in the Howe type. This truss had the great advantage of making the longest braces (of iron) resist tension only, while the shorter vertical braces resist compression. As a partially timber bridge it could not compete with the Howe truss, because it contained materially more iron and consequently was more costly. This structure practically closed the period of development of timber bridges.

72. Squire Whipple's Work.—What amounted to a new epoch in the development of bridge construction in this country practically began in 1840 when Squire Whipple built his first bowstring truss with wrought-iron tension and cast-iron compression members. While the Pratts and Howe had begun to employ to some extent the analysis of stresses in the design of their bridge members, the era of exact bridge analysis began with Squire Whipple. He subjected his bridge designs to the exacting requirements of a rational analysis, and to him belongs the honor of placing the design of bridges upon the firm foundation of a systematic mathematical analysis.

73. Character of Work of Early Builders.—The names of Palmer, Burr, and Wernwag were connected with an era of admirable engineering works, but, with bridge analysis practically unknown, and with the simplest and crudest materials at their disposal, their resources were largely constituted of an intuitive engineering judgment of high quality and remarkable force in the execution of their designs never excelled in American engineering. They occasionally made failures, it is true, but it is not recorded that they ever made the same error twice, and the works which they constructed form a series of precedents which have made themselves felt in the entire development of American bridge building.

CHAPTER VII.

74. Modern Bridge Theory.—The evolution of bridge design having reached that point where necessity of accurate analysis began to make itself felt, it is necessary to recognize some of the fundamental theoretical considerations which lie at the base of modern bridge theory, and which involve to a considerable extent that branch of engineering science known as the elasticity or strength of the materials used in engineering construction.

The entire group of modern bridge structures may be divided into simple beams or girders, trusses, arches, suspension bridges, and arched ribs, each class being adapted to carry either highway or railway traffic. That class of structure known as beams or girders is characterized by very few features. There are solid beams like those of timber, with square or rectangular cross-sections, and the so-called flanged girders which are constituted of two horizontal pieces, one at the top and the other at the bottom, connected by a vertical plate running the entire length of the beam. The fundamental theory is identically the same for both and is known as the "common theory of flexure," i.e., the theory of beams carrying loads.

If an ordinary scantling or piece of timber of square or rectangular cross-section, like a plank or a timber joist, so commonly used for floors, be supported at each end, it is a matter of common observation that it will sustain an amount of load depending upon the dimensions of the stick and length of span. When such a bar or piece is loaded certain forces or stresses, as they are called, are brought into action in its interior. The word "stress" is used simply to indicate a force that exists in the interior of any piece of material. It is a force and nothing else. It is treated and analyzed in every way precisely as a force. If the stresses or forces set up by the loading in the interior of the bar become greater than the material can resist, it begins to break, and the breaking of that portion of the timber in which the stresses or forces are greatest constitutes its failure. The load which produces this failure in a beam is called the breaking load of the beam. In engineering practice all beams are so designed or proportioned that the greatest load placed on them shall be only a safe percentage of the breaking load; the safe load usually being found between $\frac{1}{3}$ and $\frac{1}{6}$ of the breaking load. In most buildings the safe or working load, as it is called, is probably about $\frac{1}{4}$ of the breaking load.

75. The Stresses in Beams.—The proper design of beams or girders to carry prescribed loads is based upon the stresses which are developed or brought into action by them. It can easily be observed that if a beam supported at each end be composed of a number of thin planks or boards placed one upon the other, it will carry very little load. Each plank or board acts independently of the others and a very small load will cause a sag, as shown in Fig. 6. If there be taken, on the other hand, a beam made of a single stick of timber of the same width and depth as the number of planks shown in Fig. 6, so as to secure the solid beam shown in Fig. 7, it is a further common observation that this latter beam may carry many times the load which the laminated beam, shown in Fig. 6, sustains. The thin planks or boards readily slide over each other, so that the ends present

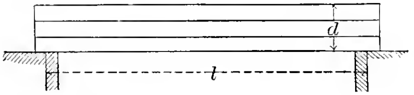


FIG. 5.

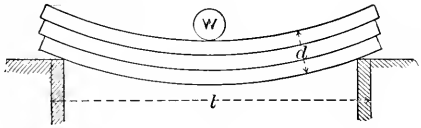


FIG. 6.

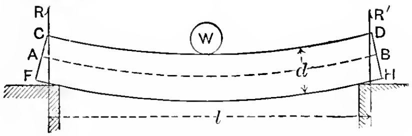


FIG. 7.

the serrated form shown in Fig. 6. The preventing of this sliding is the sole cause of the greatly increased stiffness of the solid beam shown in Fig. 7, for there is thus developed along the imaginary horizontal sections in the solid beam of Fig. 7 what are called shearing forces or stresses; and since they exist on horizontal sections or planes running throughout the entire length of the beam, they are called horizontal shears.

At each end of the beam shown in Fig. 7 there will be an upward or supporting force exerted by the abutments on which the ends of the beam rest. Those upward or supporting forces are shown at *R* and *R′* and are called reactions, because the abutments, so to speak, react against the ends of the beam when the latter is loaded. These reactions depend for their value on the amount and the location of the loading which the beam carries. Obviously these upward forces or reactions tend to cut or shear off the ends of the beam immediately above them, and if the loads were sufficiently large and the beam kept from bending, the reactions would actually shear off those ends, just as punches or shears in a machine-shop actually shear off the metal when the rivet-hole is punched, or when a plate is cut by shearing into two parts. The beam, however, bends or sags before shearing apart actually takes place.

76. Vertical and Horizontal Shearing Stresses.—If it be supposed that the length of the beam is divided into a great number of parts by imaginary vertical lines, like those shown in Fig. 8, then vertical shearing forces will be developed in those vertical planes and sometimes, though not often, they are enough to

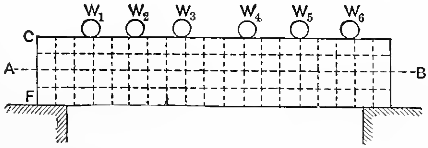


FIG. 8.

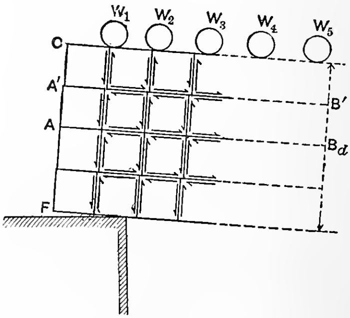


FIG. 9.

cause failure. It is not an uncommon thing, on the other hand, in timber to have actual shearing failure take place along a horizontal plane through the centre of the beam. Indeed this is recognized frequently as the principal method of failure in very short spans. When this horizontal shearing failure takes place, the upper and lower parts of the beam slide over each other and act precisely like the group of planks shown in Fig. 6.

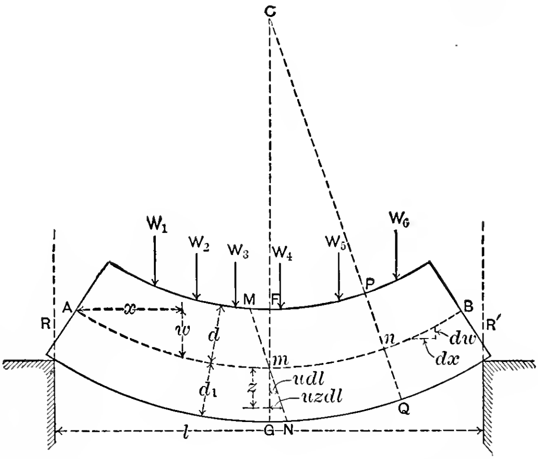
If, then, the loaded beam be divided by vertical and horizontal planes into the small rectangular portions shown in Figs. 8 and 9, on each such vertical and horizontal imaginary plane there will be respectively vertical and horizontal shearing forces, which are shown by arrows in Fig. 9. It will be noticed in that figure that in each corner of the rectangle the two shearing forces act either toward or from each other; in no case do the two adjacent shearing forces act around the rectangle in the same direction. This is a condition of shearing stresses peculiar to the bent beam. It can be demonstrated by theory and is confirmed by experiment. There is a further peculiarity about these shearing forces which act in pairs either toward or from the same angle in any rectangle, and it is that the two stresses adjacent to each other have precisely the same value per square inch (or any square unit that may be used) of the surface on which they act. These stresses per square inch vary, however, either along the length of the beam or as the centre line of any normal cross-section is departed from. They are greatest along the centre line or central horizontal plane represented by AB, and they are zero at the top and bottom surfaces of the beam.

Inasmuch as the horizontal shear along the plane $A'B'$ is less than that along AB in Fig. 9, a part of the latter has been taken up by the horizontal fibres of the beam lying between the two planes. In other words, the horizontal layer of fibres at $A'B'$ is subjected to a greater stress or force along its length than at AB. The same general observation can be made in reference to any horizontal layer of fibres that is farther away from the centre than another. Hence the farther any fibre is from the centre the greater will be the stress or force to which it is subjected in the direction of its length. It results, then, that the horizontal layers of fibres which are farthest from the centre line of the beam, i.e., those at the exterior surfaces, will be subjected to the greatest force or stress, and that is precisely what exists in a loaded beam whatever the material may be.

77. Law of Variation of Stresses of Tension and Compression.—Since a horizontal beam supported at each end is deflected or bent downward when loaded, it will take a curved form like that shown in either Fig. 7 or Fig. 10; but this deflection can only take place by the shortening of the top of the beam and the lengthening of its bottom. This shows that the upper part of the beam is compressed throughout its entire length, while the lower part is stretched. In engineering language, it is stated that the upper part of the beam is thus subjected to compression and the lower part to tension. The horizontal layers or fibres receive their tension and compression from the vertical and horizontal shearing forces in the manner already explained. If the conditions of loading of the bent beam should be subjected to mathematical analysis, it would be found that throughout the originally horizontal plane *AB*, Fig. 7, passing through the centre of each section there would be no stress of either tension or compression, although the horizontal shearing stress there would be a maximum. Further, as this central plane is departed from the stress of tension or compression per square inch in any vertical section would be found to increase directly as the distance from it. This is a very simple law, but one of the greatest importance in the design of all beams and girders, whatever may be the form or size of cross-section. It is a law, which applies equally to the solid timber beam and to the flanged steel girder, whether that girder be rolled in the mill or built up of plates and angles or other sections in the shop. It is a fundamental law of what is called the common theory of flexure, and is the very foundation of all beam and girder design. The horizontal plane represented by the line *AB* in Fig. 8, along which there is neither tension nor compression, is called the "neutral plane," and its intersection with any normal cross-section of the beam is called the "neutral axis" of that section. Mathematical analysis shows that the neutral plane passes through the centres of gravity of all the normal sections of the beam and, hence, that the neutral axis passes through the centre of gravity of the section to which it belongs.

78. Fundamental Formulæ of Theory of Beams.—The fundamental formulæ of the theory of loaded beams may be quite simply written. Fig. 10 exhibits in a much exaggerated manner a bent beam supporting any system of loads W_1, W_2, W_3, etc.,

FIG. 10.



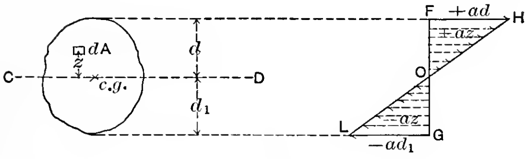


FIG. 11.

while Fig. 11 shows a normal cross-section of the same beam. In Fig. 10 *AB* is the neutral line, and in Fig. 11 *CD* is the neutral axis passing through the centre of gravity, *c.g.*, of the section.

If a is the amount of force or stress on a square inch (or other square unit), i.e., the intensity of stress, at the distance of unity from the neutral axis *CD* of the section, then, by the fundamental law already stated, the amount acting on another square inch at any other distance z from the neutral axis will be az. This quantity is called the "intensity of stress" (tension or compression) at the distance z from the neutral axis. Evidently it has its greatest values in the extreme fibres of the section, i.e., ad and ad_1. At the neutral axis az becomes equal to zero. FG in Fig. 11 represents the same line as FG in Fig. 10. If the line FH in Fig. 11 be laid down equal to ad and at right angles to FG, and if O represent the centre of gravity, *c.g.*, of the section, then let the straight line LH be drawn. Any line drawn parallel to FH from FG to LH will represent the intensity of stress in the corresponding part of the beam's cross-section. Obviously, as these lines are drawn in opposite directions from FG, those above O will indicate stress of one kind, and those below that point stress of another kind, i.e., if that above be tension, that below will be compression. It can be demonstrated by a simple process that the total tension on one side of the neutral axis is just equal to the total compression on the other side, and from that condition it follows that the neutral axis must pass through the centre of gravity or centroid of the section.

Returning to the left-hand portion of Fig. 11, let dA represent a very small portion of the cross-section; then will $az.dA$ be the amount of stress acting on it. The moment of this stress or force about the neutral axis will be $azdA.z=az^2.dA$. If this expression be applied to every small portion of the entire section, the aggregate or total sum of the small moments so found will be the moment of all the stresses in the section about the neutral axis. That moment will have the value

$$M=\int az^2.dA=a\int z^2d.A=aI. \quad . \quad . \quad . \quad . \quad (1)$$

In equation (1) the symbol $\int$ means that the sum of all the small quantities to the right of it is taken, and I stands for that sum which, in the science of mechanics, is called the moment of inertia of the cross-section about its neutral axis. The value of the quantity I may easily be computed for all forms of section. Numerical values belonging to all the usual forms employed in engineering practice are found in extended tables in the handbooks of the large iron and steel companies of the country, so that its use ordinarily involves no computations of its value.

Equation (1) may readily be changed into two other forms for convenient practical use. In Fig. 10 mn is supposed to be a very short portion of the centre line of the beam represented by dl. Before the beam is bent the section FG is supposed to have the position MN parallel to PQ. Also let u be the small amount of stretching or compression (shortening) of a unit's length of fibre at unit's distance from the centre line AB of the beam, then will udl and $uzdl$ be the short lines parallel to GN in the triangle GmN shown in the figure. The point C is the centre of curvature of the line mn, and $Cn = Cm$ is the radius. The two triangles Cnm and mNG are therefore similar, hence

$$\frac{udl}{1} = \frac{mn}{\rho} = \frac{dl}{\rho}; \quad \therefore u = \frac{1}{\rho}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the quantity called the coefficient or modulus of elasticity be represented by E, then, by the fundamental law of the theory of elasticity in solid bodies,

$$a = Eu. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

As has already been shown, the greatest stresses (intensities) in the section are $+ad$ (tension) and $-ad_1$ (compression). If K represent that greatest intensity of stress, then

$$K = ad, \quad \text{and} \quad a = \frac{K}{d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If the value of a from equation (4) be substituted in equation (1),

$$M = \frac{KI}{d}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

79. Practical Applications.—Equation (5) is a formula constantly used in engineering practice. All quantities in the second member are known in any given case. K is prescribed in the specifications, and is known as the "working resistance" in the design of beams and girders. For rolled steel beams in buildings it is frequently taken at 16,000 pounds, i.e., 16,000 pounds per square inch, about one fourth the breaking strength of the steel. In railroad-bridge work it may be found between 10,000 and 12,000 pounds, or approximately one fifth of the breaking strength of the steel. The quantities I and d depend upon the form and dimensions of the cross-section, and are either known or may be determined. The quotient $I \div d$ is now known as the "section modulus," and its numerical values for all forms of rolled beams can be found in published tables. The use of equation (5) is therefore in the highest degree convenient and practicable.

80. Deflection.—It is frequently necessary, both in the design of beams and framed bridges, to ascertain how much the given loading will cause the beam or truss to sag, or, in engineering language, to deflect below the position occupied when unloaded. The deflection is determined by the sagging in the vertical plane of the neutral line below its position when the structure carries no load. In Fig. 10 the curved line AB is the neutral line of the beam when supporting loads. If the loads should be removed, the line AB would return to a horizontal position. The line drawn horizontally through A and indicated by x is the position of the centre line of the beam before being bent. The vertical distance w below this horizontal line shows the amount by which the point at the end of the line x is dropped in consequence of the flexure of the beam. The vertical distance w is therefore called the deflection. Evidently the deflection varies with the amount of loading and with the distance from the end of the beam. The curved line AB in one special case only is a circle. The general character of that curve is determined by the loading and the length of span.

In order that the deflection may be properly considered it is necessary that the relation between x and w shall be established for all conditions of loading and length of span. If the value of u from equation (2) be placed in equation (3), there will result

$$a = \frac{E}{\rho}. \qquad (6)$$

If the value of a from equation (6) be substituted in the last member of equation (1), there will at once result

$$M = \frac{EI}{\rho}. \qquad (7)$$

It is established by a very simple process in differential calculus that

$$\frac{1}{\rho} = \frac{d^2w}{dx^2}. \qquad (8)$$

Hence, substituting from equation (8) in equation (7),

$$M = EI\frac{d^2w}{dx^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

Equation (9) may be used by means of some very simple operations in integral calculus to determine the value of w in terms of x and the loads on the beam when the value of the bending moment M is known, and the procedures for determining that quantity will presently be given.

Using the processes of the calculus, the two following equations will immediately be found:

$$\frac{dw}{dx} = \frac{1}{EI}\int Mdx; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

$$w = \frac{1}{EI}\int\int Mdx^2. \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

As already explained, numerical values for both E and I may be taken at once from tables already prepared for all materials and for all shapes of beams ordinarily employed in structural work, so that equation (11) enables the deflection or sag of the bent beam to be computed in any case. The expression $\frac{dw}{dx}$ is the tangent of the angle made by the neutral line of a bent beam with a horizontal line at any given point, and it is a quantity that it is sometimes necessary to determine. dw and dx are indefinitely short vertical and horizontal lines respectively, as shown immediately to the left of B in Fig. 10.

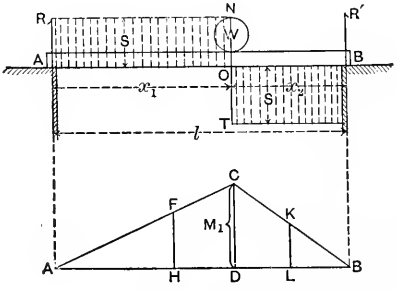
Equation (11) is not used in structural work nearly as much as equation (5), but both of them are of practical value and involve only simple operations in their use.

81. Bending Moments and Shears with Single Load.—The second members of equations (5) and (9) exhibit values of the moments of the internal forces or stresses in any normal cross-section of a bent beam about the neutral axis of the section, while the values of M must be expressed in terms of the external forces or loading. Inasmuch as the latter moment develops just the internal moment, it is obvious that the two must be equal. In order to write the value of the external moment in terms of any loading, it is probably the simplest procedure to consider a beam carrying a single load. In Fig. 12, *AB* is such a beam, and *W* is a load which may be placed anywhere in the span, whose length is *l*. The distances of the load from the abutments are represented by x_1 and x_2. The reactions or supporting forces exerted under the ends of the beam at the abutments are shown by *R* and *R′*. The reactions, determined by the simple law of the lever, are

$$R = W\frac{x_2}{l} \quad \text{and} \quad R' = W\frac{x_1}{l}. \qquad (12)$$

The greatest bending moment in the beam will occur at the point of application of the load, and its value will be

$$M_1 = Rx_1 = W\frac{x_1 x_2}{l} = -R'x_2. \qquad (13)$$



$$M_1 = Rx_1 = -R'x_2 = \frac{Wx_1x_2}{l}$$

FIG. 12.

The bending moments at the end of the beam are obviously zero, and the second and fourth members of equation (13) show that the moment increases directly as the distance from either end. Hence in the lower portion of Fig. 12, at *D*, immediately under the load *W*, the line *DC* is laid off at any convenient scale to represent the moment M_1. The straight lines *AC* and *CB* are then drawn. Any vertical intercept, as *FH* or *KL*, between *AB* and either *AC* or *CB* will represent the bending moment at the corresponding point in the beam. The simple triangular diagram *ACB* therefore represents the complete condition of bending of the beam under the single load *W* placed at any point in the span.